A Noisy Model of Individual Behaviour

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1 INTRODUCTION

Past decades have witnessed growing empirical evidence that calls into question the utility maximization paradigm. For a description of systematic errors made by experimental subjects, see Arkes and Hammond (1986), Hogarth (1980), Kahneman, Slovic, and Tversky (1982), and Nisbett and Ross (1980), and the survey papers by Payne, Bettman, and Johnson (1982), and by Pitz and Sachs (1984).

An important question is whether the observed deviations from rationality are important for economic behavior. To argue for the importance of these deviations, one should either provide applicable examples where these deviations are observed in economic environments, or provide theoretical examples where a small amount of non-maximizing behavior may have large effects on the equilibrium.

On the empirical side, there is a large body of evidence cited in Kagel and Roth's (1995) review of existing empirical studies of auctions. They describe systematic deviations from the predictions of the standard theory. Since auctions represent one of the most important forms of market organization (as may be evident, for example, from their role in facilitating the privatization process in the democracies of Eastern Europe and also in Western countries), this provides one example of an economically significant violation of the rationality paradigm.

There also exists a large body of theoretical literature that can be brought to bear upon the importance of deviations from full rationality. For example, Kandori, Mailath, and Rob (1993) developed an evolutionary model of coordination, in which a small mutation rate may prevent the system from getting stuck at an equilibrium which is risk dominated. Similar results were obtained by Fudenberg and Harris (1992), Young (1993). Selten (1975) argued that a small amount of noise can be used to rule out certain Nash equilibria. In a general equilibrium context, Akerlof and Yellen (1985) showed that small deviations from rationality may have a significant impact upon equilibrium outcomes and the welfare of agents.

For a comprehensive review of the importance of bounded rationality in economic models, see Conlisk (1996). This paper also contains an extensive literature survey. Learning is an important component of boundedly rational behavior. There exists an extensive body of psychological learning literature. See, for example, Estes (1950), Bush and Mosteller (1955). These models were introduced into economics by Luce (1959). In his model, the probability of choosing a particular alternative j from a finite set M, is given by the formula

$$prob(j) = \frac{V_j}{\sum_{h \in M} V_h}$$

where the function $V : M \to R$ is called a scaling function. Consider the function $U = \ln V$. In terms of the function U, the probability distribution above is the logistic one; and the function U can be interpreted as a utility function.

Since importance of learning is also well understood in economic literature, it may therefore be interesting to put together ideas of learning and randomness. Such an attempt was made by Anderson, Goeree and Holt (1997). They presented a dynamic model in which agents adjust their decisions in the direction of higher payoffs, subject to random error. This process produces a probability distribution over players, decisions whose time evolution follows the Fokker-Planck equation. The authors found a steady state of the process and proved that it is globally stable. However, they derived their equations only for one-dimensional case and concentrate on i.i.d. errors. I will provide generalization of their models for multidimensional case and discuss qualitatively what would change if errors are autocorrelated.

The model provides both justifications and limitations for the use of the logit model, popular in the empirical Industrial Organization literature. More precisely, one can obtain restrictions on the covariance matrix of mistakes that are consistent with a logit distribution being a steady state of the model.

The logit model has some well-recognized difficulties (Luce 1959). Consider, for example, a situation when an individual has to choose between driving to work or taking a bus. Suppose she slightly prefers to take a bus. Then the logit model will suggest that the probability of driving is slightly below 1/2. Now suppose, that instead of choosing between driving and taking a bus, the individual has to choose between driving, choosing a red bus, and choosing a blue bus. Assume she has no preference for color. Then, intuitively, one might expect that the probability she decides to drive will remain the same. However, the logit model predicts that now all the probabilities will be close to 1/3.

Putting the logit model into a learning context allows to shed some light on the "bus paradox." Indeed, as it will be shown below, the logit model arises as a steady state of a learning rule when mistakes of a given size in each direction are equally likely. In the context of previous example, if the individual intends to take a red bus, she should be equally likely to take a blue bus or to drive instead. This behavior does not seem plausible, hence, it is not surprising that the steady state resulting from it possesses some counterintuitive properties.

The paper is organized in the following way. Section 2 introduces a model of noisy individual adjustment. Section 3 studies the evolution of the population density. Section 4 discusses experimental data on learning of Merlo and Schotter (1999) in the context of my model. Section 5 concludes.

2 A MODEL

Assume an individual repeatedly faces with a problem of choosing an alternative from a one-connected, compact set $\Omega \subset R^n$. Consider a population consisting of a continuum of identical agents. Assume that the choices made by each agent follow the stochastic process:

$$dx = \nabla\Pi(x)dt + \Lambda(x,t)dW, \tag{1}$$

when x is an interior point of Ω. If $x \in Bd(\Omega)$ the agent follows rule (3.1) whenever possible, and stays put otherwise. The first term in (1) corresponds to a gradient dynamics and says that agents adjust their choices in the direction of the maximal increase of payoffs. The second term states that these choices are subject to random error. Functions Π and Λ are assumed to be twice continuously differentiable in the interior of the set Ω and continuously differentiable on the boundary. The vector dW is a vector of independent, standard Weiner processes. This assumption implies that errors are uncorrelated in time, though correlation among different components of x is permitted and is given by the matrix $\Gamma(x,t) = \Lambda^T(x,t)\Lambda(x,t)$. Dependence of Λ on x allows, for instance, the individual to think harder to reduce an error if the losses associated with it are bigger, while dependence of Λ on time allows the variance to change due, for instance, to learning.

This model is a particular case of the general model introduced in the previous chapter. The general model is reduced to it when information about the behavior of others is not available or is ignored. The simulated annealing procedure, frequently used in numerical analysis, can be considered as a particular case of the dynamics specified above. In simulating annealing algorithms, the system, with some probability moves in the direction of the gradient, and with some probability makes a choice at random from some distribution. The variance of this distribution decreases in time according to the exogenously specified, so-called annealing, schedule. As time goes to infinity, variance goes to zero. For a detailed discussion of the method of simulated annealing, see Laarhoven (1988). In a behavioral context, the random choice in the simulated annealing procedure can be interpreted as experimentation, and the decrease in variance as the result of learning.

3 EVOLUTION OF THE POPULATION DENSITY OF BOUNDEDLY RATIONAL AGENTS

In this section I will consider a population consisting of individuals who follow the adaptation rule (1). I assume that the set of admissible alternatives Ω is compact, one-connected, and has a smooth boundary. Let $f(x,t)$ be the density of the population that chooses x at time t and assume that the initial choices x_0 are distributed according to the function $g(x)$.

Theorem 1 *The time evolution of the function $f(x,t)$ is governed by the partial differential equation:*

$$\frac{\partial f(x,t)}{\partial t} + div(\nabla \Pi(x) f(x,t)) = \frac{1}{2} Tr(D^2(\Gamma(x,t) f(x,t))) \tag{2}$$

and satisfies the boundary condition

$$< \nabla \Pi(x), n(x) > f - \frac{1}{2} < \Gamma(x,t) \nabla f, n(x) >= 0 \tag{3}$$

and the initial condition $f(x,0) = g(x)$*, where the vector* $n(x)$ *is the unit vector normal to the boundary of the set* Ω *at each point* x *of the boundary.*

3.1 A ONE-DIMENSIONAL CASE

Consider a particular case of the model when the choice space is a segment $[a_1, a_2]$ of the real line and $\Gamma(x,t) = \sigma^2$ is constant. This is the case originally considered by Anderson, Goeree and Holt (1997). The unique stationary solution of the problem (2)-(3) in this case has the form:

$$f(x) = \frac{\exp(\frac{2\Pi(x)}{\sigma^2})}{\int_{a_1}^{a_2} \exp(\frac{2\Pi(y)}{\sigma^2}) dy} \tag{4}$$

This distribution is known as the logit distribution. Note that this is the same type of distribution as obtained by Luce (1959) from the axiomatic approach. It possesses an interesting property called Independence of Irrelevant Alternatives (IIA).

Definition 1 *A population density* $f(x)$ *satisfies IIA if for any* x_1, $x_2 \in \Omega$, *the ratio $f(x_1)/f(x_2)$ does not depend on Ω.*

The IIA property applied to stationary choice densities says that the ratio of the probability that the choice is in an $\varepsilon-$ ball centered at the point x_1 to the probability that it is in an $\varepsilon-$ ball centered at point x_2 does not depend on whether some other choice z is available, up to the order $o(\varepsilon)$. For the density function (3.4), the ratio $f(x_1)/f(x_2)$ depends only on the payoff difference at points x_1 and x_2 and, hence, satisfies IIA.

Before going further it is interesting to discuss the effect of the autocorrelated. experimentation on the equation governing the time evolution of the density function. Assume that the adaptation rule for the variable x is given by:

$$dx = \Pi'(x)dt + dz \tag{5}$$

where dz follows the stochastic differential equation:

$$dz = -\alpha z dt + \sigma dW \tag{6}$$

with $z(0) = z_0$. This is called the Ornstein-Uhlenbeck process.

Theorem 2 *If the adaptation rule for variable x is given by (3.5), $F(x,t)$ is* *the probability that the choice variable is less than x at instant t, and $f(x,t)$ is the corresponding density function, then $f(x,t)$ solves the following:*

$$\frac{\partial f(x,t)}{\partial t} + \frac{\partial(\Pi'(x)f(x,t)}{\partial x} = (\frac{\sigma^2}{2} + z_0 e^{-\alpha t})\frac{\partial^2 f(x,t)}{\partial x^2} \tag{7}$$

$$\Pi'(a_k)f(a_k,t) - (\frac{\sigma^2}{2} + z_0 e^{-\alpha t})f'(a_k,t) = 0 \tag{8}$$

for every t and $k = 1, 2$, and

$$f(x,0) = g(x) \tag{9}$$

For a proof see, for example, Bhattacharya and Waymire (1990). Let us next require in addition that the error process be stationary. This is equivalent to requiring $\alpha \geq 0$. To see this note that the Ornstein-Uhlenbeck process is a continuous time version of a first order autoregressive process. If one considers observations of z made at discrete instants $k\Delta t$, where k is natural number and Δt is sufficiently small, then z will follow a discrete time first order autoregressive process:

$$z_{t+1} = \rho z_t + \varepsilon_t \tag{10}$$

where $\rho = \exp(-\alpha \Delta t)$ and ε_t is white noise. A first order autoregressive process is known to be stationary if $|\rho| < 1$, which justifies the above assertion.

For the case of a stationary error process, equations (7) and (8) imply that autocorrelation of errors will not have long run effects. Nevertheless, its short run effect can be quite significant. For example, if z_0 is such that the diffusion term in (7) vanishes at $t = 0$, then for small t, the individual will approximately follow the gradient dynamics, which would result in a rapid decrease of variance and might create an illusion that the individuals learned the rational choice. However, for larger t, one will observe a divergence of the distribution from a nearly rational outcome towards a more dispersed logit outcome. This might be interpreted as an exacerbation of errors by learning. A discussion of the possibility of this effect, a description of alternative explanations, and related literature can be found in Baumann et al. (1991).

3.2 A MULTIDIMENSIONAL CASE

Having discussed in detail the one-dimensional case, let us return to the multidimensional setting, that is, assume that the choice space Ω is a compact, one-connected subset of R^L with smooth boundary, where $L > 1$.

Theorem 3 *Assume that $\Gamma(x,t)$ does not depend on t. Then a steady state solution of problem (2)-(3) exists. It is unique and asymptotically stable.*

For a proof, see Ito (1979). This result generalizes the analogous result for the one-dimensional case obtained by Anderson et al. (1997). Nevertheless, there is a profound difference between the one-dimensional and multidimensional cases. In the one-dimensional case, the steady state satisfies the IIA property, which might be economically significant at least in some contexts. It turns out that in the multidimensional case, this property is not satisfied unless rather strong assumptions either on errors or on the payoff structure are imposed.

To see this, introduce a vector j by the formula:

$$j(x) = -\nabla\Pi(x)f(x) + \frac{1}{2}\Gamma(x)\nabla f(x). \tag{11}$$

Let $n(x)$ be a unit vector, normal to the boundary $\partial\Omega$ of the choice set. Then in the steady state, $j(x)$ should solve the following boundary problem

$$div(j(x)) = 0 \tag{12}$$

$$\langle j(x), n(x)\rangle = 0 \tag{13}$$

for any $x \in \partial\Omega$.

The distribution f is then determined by the system of first-order partial differential equations:

$$j(x) = -\nabla\Pi(x)f(x) + \frac{1}{2}\Gamma(x)\nabla f(x). \tag{14}$$

The IIA property implies that a change in Ω will result in multiplication of f, and hence of j, by a constant, that is $j_{new} = Cj_{old}$[1]. Hence j_{new} should solve the same boundary problem, but on a different domain. The only vector j that would solve (3.12)-(3.13) for any domain is $j = 0$. Hence IIA together with the definition of j implies that $f(x,t)$ solves the system

$$\nabla\Pi(x)f - \frac{1}{2}\Gamma(x)\nabla f(x,t) = 0. \tag{15}$$

Young's theorem implies that the necessary and sufficient condition for (3.15) to have a solution is

$$\sum_k [(\frac{\partial\Gamma^{-1}_{ik}}{\partial x_j} - \frac{\partial\Gamma^{-1}_{jk}}{\partial x_i})\frac{\partial\Pi}{\partial x_k} + (\Gamma^{-1}_{ik}\frac{\partial^2\Pi}{\partial x_k\partial x_j} - \Gamma^{-1}_{jk}\frac{\partial^2\Pi}{\partial x_k\partial x_i})] = 0$$

[1] This relation should hold at each point which belongs to the intersection of the new and the old choice space.

If the errors are independent of x, then this condition implies either that errors are spherical ($\Gamma(x) = \sigma^2 I$), or the payoff function is additively separable in different components of the choice vector x and the matrix Γ is diagonal. Hence, in the multidimensional case IIA holds only under strong conditions on the errors or on the payoff function.

Although very special, the assumption of spherical errors may still be reasonable in some situations. For example, in a political-economic context when x is a vector of the characteristics of a candidate for office. In this case, some authors have formalized the preferences of a voter as the negative of the Euclidean distance from the candidate's characteristics vector to the bliss point of the voter. This approach is used, for example, in the papers of Tullock (1967) and Caplin and Nalebuff (1988). Another example of a situation where this assumption is relevant comes from spatial competition models in the theory of industrial organization. In these models, firms sell identical goods at different locations. A consumer's utility is the negative of the price paid for the good minus the Euclidean distance between her location and the location of a firm. Popular models of this kind are the model of a linear city developed by Hotelling (1929), and the model of a circular city developed by Salop (1979).

Returning to the "bus paradox" described in Introduction, note that it can be put in a continuous choice context introducing two variables: "busness" (it measures similarity of a vehicle to a bus. For example, for a car it is), for a mini-van 1/2, and for a bus 1), and color (the wavelength of light). Then the problem of choosing between driving and taking a bus can be imbedded into a problem of choosing a vehicle out of a continuous two-parametric family. For the steady state choice density to satisfy IIA, mistakes in the direction of color should have the same distribution as in the direction of "busness," which is implausible.

4 APPLICATION OF THE THEORY TO EXPERIMENTAL DATA.

In this section I use the data obtained in the Merlo and Schotter (1999) experiments to test the model developed in this chapter. Merlo and Schotter perform experiments of a tournament variety. These experiments are similar to those of Bull, Schotter and Weigelt (1987) and Schotter and Weigelt (1992). In these experiments, randomly paired subjects must in each round choose a natural number e between 0 and 100. After the numbers are chosen, two random numbers are generated independently from a uniform distribution on the segment $[-a, a]$. The first of these random numbers is added to the number chosen by the first player, while the second one is added to the number chosen by the second player. The player with the higher sum wins a prize M and the other player wins a prize m. The monetary payoff is determined by the prize minus the "cost" associated with the number chosen by the individual, given by $e^2/2k$. Merlo and Schotter used the following values for parameters: $k = 500$, $a = 40$, $M = 29$, and $m = 17.2$. In this case, assuming risk neutrality, the game has a unique symmetric Nash equilibrium $e = 37$. The authors programmed a computer to play its part of the Nash equilibrium. The subjects knew that they are facing a computer that was programmed to play $e = 37$. Hence, the problem was basically one of individual decision making.

Merlo and Schotter (1999) distinguished between two environments: Learn-While-You-Earn (LWYE) and Learn-Before-You-Earn (LBYE). The game lasted for a known number of periods T. In the actual experiments $T = 75$. In the LWYE environment players got at the end of the game the sum (possibly discounted) of their payoffs earned in each round. In LBYE experiments they did not get real payoffs for round $1 - 74$ but observed what they would have gotten if the game had been real. In round 75 they got their payoff. Stakes in this round were 75 times higher than in the games of the LWYE environment.

The authors concluded that in the LBYE environment the choices of the agents in the last round were closer to the optimal value 37. They explain this by noting that these subjects experimented more aggressively during previous periods since they did not incur any cost by experimentation. Though this conclusion seems reasonable, one might also question whether 37 is really the optimal choice, that is, whether risk neutrality is a reasonable assumption. Nevertheless, the conclusion that more learning has occurred in the LBYE environment than in the LWYE environment can be given a precise meaning in the terms of the model of this chapter. This version of the general model seems appropriate here since the subjects did not observe the choices of other subjects, hence there was no scope for social adaptation. Under the assumptions on the parameters of the model, the probability to win if the announced number is e is $(e + 43)/160$. Hence the expected utility from choice e is

$$\frac{e+43}{160}u(29-\frac{e^2}{1000})+\frac{117-e}{160}u(17.2-\frac{e^2}{1000}),$$

where u is the subject's Bernoulli utility function. Assume that $\Gamma(e,t) = \sigma^2$, then formula (3.4) implies that the steady state density function is given by:

$$
\begin{aligned}
f(e) &= \frac{1}{C}\exp[\frac{2}{\sigma^2}(\frac{e+43}{160}u(29-\frac{e^2}{1000})+\frac{117-e}{160}u(17.2-\frac{e^2}{1000}))], \quad (16) \\
C &= \int_0^{100}\exp[\frac{2}{\sigma^2}\{\frac{e+43}{160}u(29-\frac{e^2}{1000})+\frac{117-e}{160}u(17.2-\frac{e^2}{1000})\}]de \quad (17)
\end{aligned}
$$

One can observe that for typical choices the term $e^2/1000$ is much less then 17.2, hence one can approximate the utility function by its first order Taylor expansion. This would imply that the steady state distribution is approximately normal (the term $(u'(29) - u'(17.2)) * e^3/160,000$ is small for the typical values of e observed in the experiment assuming the agents are not too risk averse and can be neglected).

I tested the normality assumption for both environments. The extreme observations $e = 0$ and $e = 100$ were excluded from the sample on the grounds that the subjects who made these choices seemed to be ruled by principles different from those of the entire population. The choice $e = 0$ can be justified by an extreme risk aversion, and the choice $e = 100$ by a desire to win at any cost. I could not reject the normality hypothesis in the case of

LBYE experiment at the 10% significance level. For the LWYE experiment normality, can be rejected at the 47% significance level.

Using the histogram of round 75 as a proxy for the function $f(e)$, I tried to fit a quadratic model to logarithm of $f(e)$. The results of the regression are the following:

$$\log f(e) = \underset{-15.41593}{-0.770403} + \underset{19.31326}{0.855871}e - \underset{-10.29404}{0.076451}e^2 (LBYE) \tag{18}$$

$$\log f(e) = \underset{-3.901472}{-5.903613} + \underset{4.263681}{0.256824}e - \underset{-4.154517}{0.002305}e^2 (LWYE) \tag{19}$$

The numbers below the regression coefficients are the corresponding t-statistics. The quadratic function fits $\log f(e)$ extremely well for the LBYE environment: $\overline{R}^2 = 0.997772$. For the LWYE environment, the goodness of fit is much worse: $\overline{R}^2 = 0.705113$. Hence, I conclude that in the LBYE environment the players reached the steady state in 75 rounds, while in the LWYE environment they did not. Since in the LBYE environment agents seem to reach steady state, the results of this regression can be used to test the risk neutrality hypothesis. To do this I estimate the value of $M - m$ from the data, assuming risk neutrality, and then compare it with its actual value.

The estimated value is 3.6 with 10% precision. This number is very far at odds with the actual number 11.8, hence the risk neutrality hypothesis can be firmly rejected.

Now let us examine the claim of Merlo and Schotter (1999) that more aggressive sampling produces faster learning. In my model more aggressive sampling means higher σ. I will argue that the higher is σ, the faster is convergence to the steady state. Indeed, consider the solution to the problem:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial e}(\Pi'(e)f) + \frac{\sigma^2}{2}\frac{\partial^2 f}{\partial e^2} \tag{20}$$

with the initial condition $f(e,0) = g(e)$ and the boundary condition:

$$\Pi'(e)f(e) + \frac{\sigma^2}{2}f'(e) = 0 \text{ for } e = 0, 100. \tag{21}$$

Here the payoff function $\Pi(e)$ is given by:

$$\Pi(e) = \frac{e+43}{160}u(29 - \frac{e^2}{1000}) + \frac{117-e}{160}u(17.2 - \frac{e^2}{1000}). \tag{22}$$

For a general discussion of the method I use below see, for example, Hilbert and Courant (1953, Chapter VII). One can also find there a proof of all assertions made from this point to the end of this chapter.

Define ϕ_n and λ_n to be the eigenfunctions and the eigenvalues of differential operator L defined on a set of twice continuously differentiable functions on $[0, 100]$ and satisfying boundary conditions (24) by

$$Lh = -\frac{\partial}{\partial e}(\Pi'(e)h) + \frac{\sigma^2}{2}\frac{\partial^2 h}{\partial e^2}.$$

Then the general solution to the problem (23)-(24) is given by:

$$f(e,t) = \sum_{n=0}^{\infty} c_n \phi_n(e) \exp(-\lambda_n t). \tag{23}$$

The constants c_n are determined from the initial condition.

It is easy to see that $\lambda_0 = 0$ is an eigenvalue of the operator L with corresponding eigenfunction

$$\phi_0(e) = \frac{1}{C}\exp[\frac{2}{\sigma^2}(\frac{e+43}{160}u(29 - \frac{e^2}{1000}) + \frac{117-e}{160}u(17.2 - \frac{e^2}{1000}))], \tag{24}$$

$$C = \int_0^{100} \exp[\frac{2}{\sigma^2}\{\frac{e+43}{160}u(29 - \frac{e^2}{1000}) + \frac{117-e}{160}u(17.2 - \frac{e^2}{1000})\}]de \tag{25}$$

Multiplying the equation $L\phi_n = \lambda_n \phi_n$ by $\exp(-2\Pi(e)/\sigma^2)$, integrating from 0 to 100, and remembering that ϕ_n is assumed to satisfy the boundary condition

(24), one gets the following expression for λ_n:

$$\lambda_n = \frac{\int\limits_0^{100} \exp(\frac{-2\Pi(e)}{\sigma^2})(\Pi'(e)\phi_n(e) - \frac{\sigma^2}{2}\phi_n'(e))^2 de}{\int\limits_0^{100} \exp(\frac{-2\Pi(e)}{\sigma^2})\phi_n^2(e) de}. \tag{26}$$

One can see that all the eigenvalues are nonnegative, and this implies that the steady state distribution is stable. Now place the eigenvalues in ascending order. Formula (26) implies that the relaxation time, that is the time in which the density function reaches the steady state is of the order of:

$$t_{rel} = \frac{1}{\lambda_1}. \tag{27}$$

The eigenvalue λ_1 can be found as:

$$\lambda_1 = \min \int\limits_0^{100} \exp(\frac{-2\Pi(e)}{\sigma^2})(\Pi'(e)\phi(e) - \frac{\sigma^2}{2}\phi'(e))^2 de \tag{28}$$

$$s.t. \int\limits_0^{100} \exp(\frac{-2\Pi(e)}{\sigma^2})\phi(e)^2 de = 1, \quad \int\limits_0^{100} \phi de = 0. \tag{29}$$

The first order condition for problem (31)-(32) can be shown to be equivalent to the equation determining ϕ_1. Using this method one can find an estimate of λ_1 in the case of small payoff gradients. This approximation seems to be satisfied in the Merlo and Schotter (1999) experiments. To formalize it write the function $u(\cdot)$ at (25) in a form $u = \varepsilon v$, where $\varepsilon << 1$. Then

$$\lambda_1 = \frac{\pi^2 \sigma^2}{20000} + \underline{O}(\varepsilon). \tag{30}$$

Equation (33) implies that λ_1 is increasing in σ^2 and, hence, that learning time is decreasing in σ^2. Since σ^2 represents the scope of experimentation, this is in accordance with Merlo and Schotter's (1999) interpretation of their results. Formula (33) allows us to estimate σ^2 to be 27. It is worth mentioning that the value of σ^2 depends on the choice of time units. In this case a unit of time is a period of the game.

One can learn two lessons from this section. First, the model of this section provides a useful way to look at experimental data. Second, one can use the data to estimate the parameters of the model (σ in this case).

5 CONCLUSIONS

In this paper I developed a model of individual adjustment subject to mistakes. If mistake process is stationary the adjustment process converges to the unique steady state distribution. I investigated the conditions under which this distribution satisfies IIA property.

It is interesting to generalize this model to a strategic environment. A first step in this direction is undertaken by Anderson, Goeree, and Holt (1998) in their analysis of all-pay auctions. Another potentially interesting application of the model is to Spencian signalling game. Some preliminary results of the author indicate that a Pareto optimal solution can be achieved in the long-run in the case of vanishingly small noise, even if it is not a Nash equilibrium of the signalling game.

Another line of research investigates the connections of this model with a general model of adaptive behavior. For the development of these ideas see Basov (2001).

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